

# 28 | Интегриране на тригонометрични функции

$$I = \int R(\sin x, \cos x) dx \rightarrow \text{рационална функция}$$

$$\hookrightarrow t = \tan \frac{x}{2}$$

$$x = 2 \arctan t$$

$$\rightarrow \sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} = \frac{2t}{1+t^2}$$

$$\rightarrow \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} = \frac{1-t^2}{1+t^2}$$

$$\rightarrow dx = d(2 \arctan t) = \frac{2}{1+t^2} dt$$

$$\Rightarrow I = \int \underbrace{R\left(\frac{2t}{1+t^2}; \frac{1-t^2}{1+t^2}\right) \cdot \frac{2}{1+t^2}}_{\text{рационална функция}} dt$$

$$1) R(-\sin x, -\cos x) = R(\sin x, \cos x)$$

$$t = \tan x$$

$$\sin^2 x = \frac{\tan^2 x}{1 + \tan^2 x} = \frac{t^2}{1+t^2}; \cos^2 x = \frac{1}{1 + \tan^2 x} = \frac{1}{1+t^2}$$

$$\sin x \cos x = \frac{t}{1+t^2}$$

$$2) R(-\sin x, \cos x) = -R(\sin x, \cos x)$$

$\sin$  се брине за диференцирана  $\rightarrow \sin$  утврди само на реи-  
на етиенет  $\rightarrow$  изража се преко  $\cos x$ .

$$t = \cos x.$$

$$3) R(\sin x, -\cos x) = -R(\sin x, \cos x)$$

$$\hookrightarrow \text{аналогично} \rightarrow t = \sin x$$

$$I = \int R'(x) Q(x) dx = \int Q(x) d \int R'(x) dx = \int Q(x) dR(x) =$$

$$Q \in \{e, \arcsin, \arccos, \arctg, \operatorname{arccotg}\}$$

$$= Q(x)R(x) - \int R(x) dQ(x) = Q(x)R(x) - \int R(x) Q'(x) dx$$